Space and Geometry

Hussein Houdrouge -

Phil256D

September 2019

The geometry of space is a topic of a major disagreement among philosophers, physicists, and mathematicians. In this paper, I will explain the geometric differences in space between Newtonian mechanics and general relativity. Furthermore, I will discuss how we might do experiments to measure the geometry of space. Moreover, I will go through Poincare's position on geometry. Finally, I will explain the difference between substantival (absolute) and relational interpretations of spacetime, and Newton's Bucket argument in favor of the absolute space. In addition, I will argue that under current conditions, we cannot determine whether space is absolute or relational.

In the beginning, I will discuss the geometric difference between space in Newtonian mechanics and general relativity. First of all, space in Newtonian mechanics is required to be Euclidean. However, this is not the case in general relativity. Euclidean geometry is the geometry that satisfies Euclid's axioms. For instance, the sum of the three angles of a triangle is 180 degrees, or from a given point and line we can draw only one line parallel to the given line and passes through the point. In contrast, non-Euclidean geometry has a different set of axioms. In addition, it is known as curved geometry, and there are two types of curvatures. It either has positive curvature or negative curvature. For example, in positively curved geometry, the sum of the three angles of a triangle can be more than 180 degrees, and in negatively curved space, the sum of the angles can be less than 180 degrees. Furthermore, an important feature of positively curved geometry is that there are no parallel lines at all (page 67). Here, by parallel lines, I mean the geodesics; the shortest curves between two points. On the other hand, lines that follow the geodesic do not intersect in negatively curved geometry. These properties about parallel lines will give different kinds of experiences for creatures living in such geometries. For instance, in positively curved geometries, if there is an object moving far from you, you can follow it along its path; it will never disappear from your field of vision. In contrast, in negatively curved spaces, you will not be able to trace the moving object if it disappears in front of you. Thus, different sets of geometric axioms can produce different kind of observations.

Now, I will discuss how we might do experiments to measure the geometry of space. The main idea behind experiments related to measuring the geometry of space is to try and show violations or validations of the Pythagorean Theorem and the parallel postulate. If there is any violation, we can conclude that space is not Euclidean. To illustrate how the experiment goes, suppose we have three corners (objects) in space where one of them forms a 90-degree angle with the other two. We pull a cord tight between these three objects, and we send a light signal or probes to measure the length of the three sides of the formed triangle. Notice that we pull the line straight to let it follow the shortest distance between two points. After performing the measurement, we can check if the formed triangle obeys the Pythagorean Theorem. Unfortunately, this kind of experiment turns out to be logically incorrect. First, when constructing the experiment, we assumed that the shortest distance follows a straight line. This assumption is a feature of Euclidean geometry. At the end of the experiment, we conclude whether or not the space we're testing is Euclidean. Therefore, this experiment uses a circular argument; we assume that space is Euclidean to prove it Euclidean. In conclusion, we cannot rely on such an experiment because it starts with an assumption of what we're trying to prove.

In this paragraph, I will introduce the Poincare position on geometry. First of all, Poincare claimed that any experiment that managed to measure the geometry of space could be interpreted in terms of both Euclidean and non-Euclidean geometry (Page 76). Furthermore, he argued that the geometry of space is a convention. In other words, we can pick any kind of geometry and interpret the universe according to it. In this way, geometry will be like length or weight or any other properties that can be measured. Moreover, Poincare suggests that we have 'group of displacements'. These groups are the set of rules where the sequences of motion are equal to each other (Page 78). In this manner, we will have a pair of geometry and force which according to them we determine the motion of objects in space. Therefore, we can pick any pair of geometry and force and do our experiments. For instance, we can measure the length by meter or by feet, the same thing is applicable for geometry we can measure it in term of Euclidean and non-Euclidean geometry. That is, our view concerning geometry is conventional. However, a question arise counter Poincare's claim. What we are measuring when we find that the experiment verify the Pythagorean law? To answer this question, we give the example of Poincare universe. In Poincare universe there is shrinkage factor w, where the object shrink by value w between zero and one while it moves towards the edge. This shrinking factor affects the geodesics that are closer to the edge that made them look like saddle space knowing that the universe is flat plane. However, when we do an experiment that verifies the Pythagorean Theorem in the edge of this space, we are choosing the wrong curve as geodesic. Therefore, to make a precise experiment we have to know the group of displacement. And after picking a group we can perform our experiment and get the right result. Thus, Poincare claims that there is no such thing called the geometry of space, but we can make a conversion about the geometry and build our knowledge according to it.

In the following paragraph, I will explain the difference between the relational and substantival interpretation of spacetime and Newton's bucket argument. First of all, the relational interpretation, supported by Leibniz, claims that there is no such thing called space. However, all that exists are the matters or the objects, and space is the abstract logical interpretation or construction of the relation between these objects. For instance, we can make an analogy between space and family. There is no such thing called family, we cannot touch a family and apply on it some transformations. However, the family is nothing more than the set of relation between certain people. Such relations, we call them fatherhood, motherhood, and brotherhood... are what make a sense for a family. Without these relations, we cannot open a discussion about a family. Furthermore, the vacuum is conceptually possible in relational space. However, Leibniz believed God would not allow a vacuum (Page 93). On the other hand, the substantival interpretation, supported by Newton, believes that there is an absolute thing called space. However, Newton argued that the motion of bodies might be relative to other bodies, but it is also has a unique absolute place and motion (Page 94). Newton argued for absolute places and motions because he took space as something distinct from matter, something fixed where all matter exists (Page 94). In other words, space is something special, strange, immaterial, penetrable, inactive object. In order to prove his point of view, Newton produced the bucket argument. In his argument, Newton used a half-full bucket of water. When the bucket is spun, the water will spin also, and the surface of the water will become concave. In the end, the water will spin at the same speed of the bucket and it will maintain a fixed height (Page 95). Furthermore, Newton argues that the height of the water in the bucket depends on the speed of the rotation. However, the speed of the rotation is different relativity to other moving bodies. For example, a spaceship or a person on a different planet will measure the speed of rotation differently, which imply that water should have different height, but the water maintains a fixed height. This observation leads to inconsistency if we rely on relative measurements. Therefore, Newton concludes that it should be an absolute space where we can make a reference for the movement of rotating water.

Finally, I will argue that we cannot answer under the current circumstances whether space is absolute or rationalist. First, relational space has problematic in confronting the bucket experiment. Therefore, the rationalists are not able to give a complete explanation of the universe. In addition, they lack the explanation concerning the existence of vacuum; Leibniz states that God will not allow the existence of the vacuum (Page 93). Such an explanation is not enough, and anyone can argue to such metaphysical argument. For instance, someone can explain the bucket experiment metaphysically and then it will not be longer problematic for the relational. On the other hand, arguing in favor of the absolute space seems problematic. According to the absolutism, space is a strange, penetrable, inactive, immaterial object (Page 95). This view of space makes it impossible to detect. There is no way by an experiment to detect such an object. Even though space interacts act with the matter, we are not able to detect such interaction. However, the only way to prove such a thing exists could be possible using reason. For instance, Newton used the bucket experiment to prove the necessity of absolute space. But such an experiment is not sufficient to prove the existence of absolute space. In addition, this experiment assumes that Newton's law holds in the universe. But, as physics shows Newton's law has a limit so we cannot rely on them to answer such a question. In other words, our scientific knowledge is still not able to answer or explain such an experiment to draw a conclusion about the nature of space. In conclusion, argued against or with one side in not feasible, since our scientific knowledge still incomplete, and maybe both can be right and we can just assume or pick conventionally a nature of the universe and build consistent physics from it as Poincare propose for solving the problem of the geometry of space.

Last but not least, at the beginning of this paper, I explained the difference between Euclidean and non-Euclidean geometry, the geometry required by Newtonian physics and the geometry required by general relativity. Furthermore, I explained how we might do experiments in order to discover the geometry of space. In addition, I discussed whether such experiments are feasible and sufficient to measure the geometry of space. Later on, I explained Poincare's position toward geometry. In other words, I explained the conventionalism in geometry. Moreover, I ended this paper by explaining the difference between substantival space and relational space. Furthermore, I discussed Newton bucket experiment and its effect on the relational point of view. Finally, I argued that we cannot take aside in favor of any party under the current situation.

Bibliography

All the citations refer to, "Everywhere and Everywhen, Adventure in Physics and Philosophy", By Nick Huggett.